The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: February 25, 2009

Course: EE 313 Evans

Name: Set, Solution
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and homework solution sets.
- **Power off all cell phones and pagers**
- You may use any standalone calculator or other computing system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	30		Differential Equation
2	25		Convolution
3	30		Tapped Delay Line
4	15		Potpourri
Total	100		

K-36

Problem 1.1 Differential Equation. *30 points.*

For a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2}y(t)+8\frac{d}{dt}y(t)+15y(t)=x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? *5 points.*

Characteristic roots are the roots of the characteristic polynomial:

$$\lambda^2 + 8\lambda + 15 = 0$$

$$(\lambda + 3)(\lambda + 5) = 0 \qquad \text{Roots are } \lambda = -3 \text{ and } \lambda = -5$$

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. *10 points.*

$$y_0(t) = C_1 e^{-3t} + C_2 e^{-5t} \quad \text{for } t \geq 0^+$$

$$y_0'(t) = -3C_1 e^{-3t} - 5C_2 e^{-5t} \quad \text{for } t \geq 0^+$$

(c) Find the zero-input response for the following initial conditions: $y(0^+) = 0$ and $y'(0^+) = -2$. *10 points.*

$$y_0(0^+) = C_1 + C_2 = 0 \qquad \Rightarrow \qquad C_1 = -C_2$$

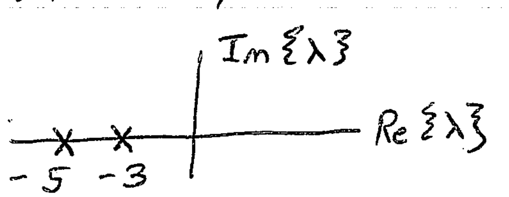
$$y_0'(0^+) = -3C_1 - 5C_2 = -2 \qquad -2C_2 = -2$$

Therefore, $C_1 = -1$ and $C_2 = 1$

$$y_0(t) = -e^{-3t} + e^{-5t} \quad \text{for } t \geq 0^+$$

(d) Is the zero-input response asymptotically stable, marginally stable, or unstable? Why? *5 points.*

The real parts of the characteristic roots are negative. Therefore, all roots are in the left-hand plane,



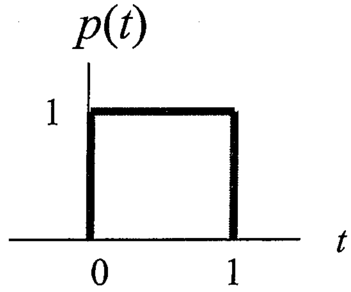
and the zero-input response is asymptotically stable.

K-37

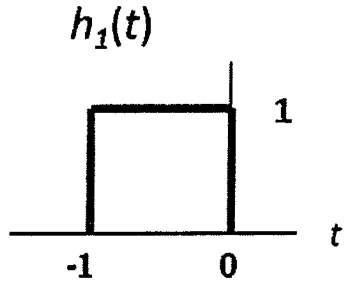
Problem 1.2 Convolution. *25 points.*

Let $p(t)$ be a causal rectangular pulse:

$$p(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$



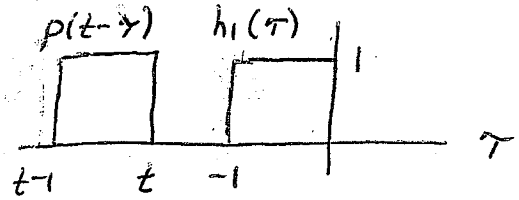
Sketch (plot) the following convolutions. On the sketches, be sure to label significant points on the horizontal and vertical axes. You do not have to show intermediate work, but showing intermediate work may qualify for partial credit.



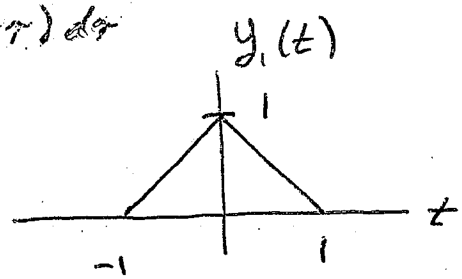
(a) Convolve $p(t)$ with $h_1(t)$ where (*10 points*)

$$h_1(t) = \begin{cases} 1 & \text{for } -1 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$$

$$y_1(t) = h_1(t) * p(t) = \int_{-\infty}^{\infty} h_1(\tau)\, p(t-\tau)\, d\tau$$

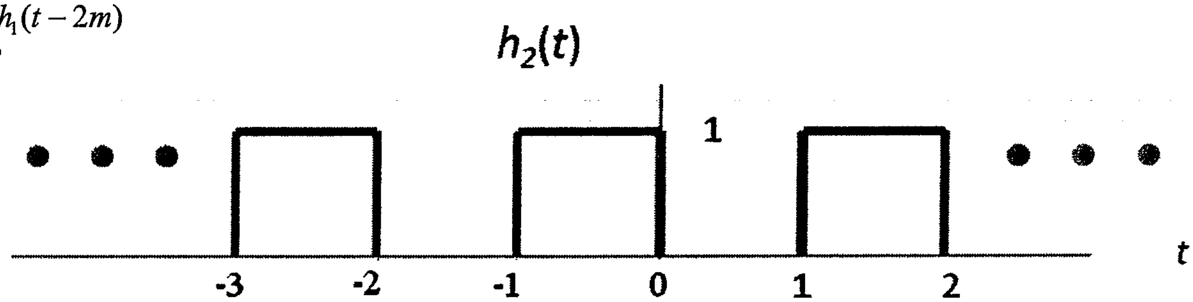


Non-zero convolution result for $-1 \le t \le 1$



(b) Convolve $p(t)$ with $h_2(t)$ where (*15 points*)

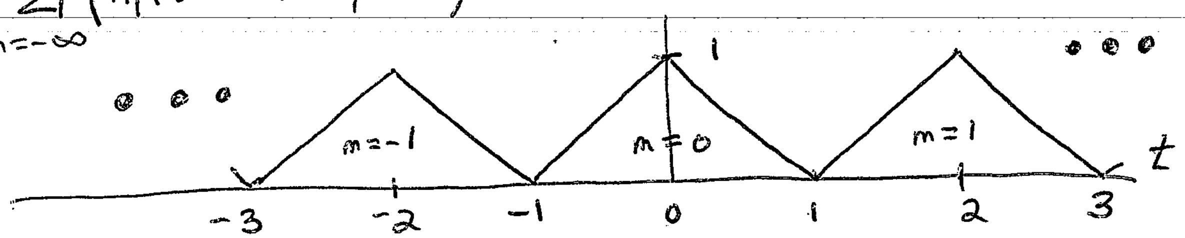
$$h_2(t) = \sum_{m=-\infty}^{\infty} h_1(t-2m)$$



$$y_2(t) = h_2(t) * p(t) = \int_{-\infty}^{\infty} h_2(\tau)\, p(t-\tau)\, d\tau$$

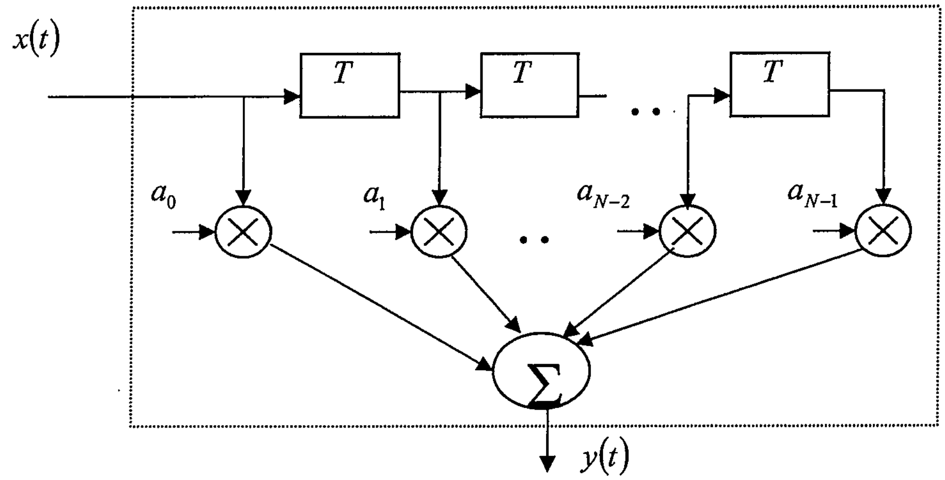
$$= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h_1(\tau-2m)\, p(t-\tau)\, d\tau = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau-2m)\, p(t-\tau)\, d\tau$$

$$= \sum_{m=-\infty}^{\infty} \big(h_1(t-2m) * p(t) \big) \Rightarrow \text{Reuse result from (a)}$$



Problem 1.3 Tapped Delay Line. *30 points.*

A linear time-invariant (LTI) continuous-time tapped delay line with input $x(t)$, output $y(t)$, and N-1 delay elements is shown below as a block diagram (from slide 2-4):



(a) What are the initial conditions? To what value(s) should they be set? *10 points.*

Each delay block initially contains a buffer of T seconds of signal. Those N-1 buffers must be zeroed out (i.e. the initial conditions must be set to zero) in order for the system to be linear and time invariant.

(b) Give a formula for the impulse response $h(t)$. *5 points.*

$$y(t) = \sum_{n=0}^{N-1} a_n \, x(t - nT)$$

Let $x(t) = \delta(t)$, $\quad h(t) = \sum_{n=0}^{N-1} a_n \, \delta(t - nT)$

(c) Is the LTI continuous-time tapped delay line always bounded-input bounded-output (BIBO) stable? Why or why not? *15 points.*

Conditional: In order for an LTI system to be BIBO stable, $\int_{-\infty}^{\infty} |h(\tau)| \, d\tau < \infty$. $\quad \int_{-\infty}^{\infty} h(\tau) \, d\tau = \int_{-\infty}^{\infty} \left(a_0 \delta(\tau) + a_1 \delta(\tau - T) + \dots \right) d\tau$

$$= a_0 + a_1 + \dots + a_{N-1}$$

$$\int_{-\infty}^{\infty} |h(\tau)| \, d\tau = \int_{-\infty}^{\infty} \left| a_0 \delta(\tau) + a_1 \delta(\tau - T) + \dots + a_{N-1} \delta(\tau - (N-1)T) \right| d\tau$$

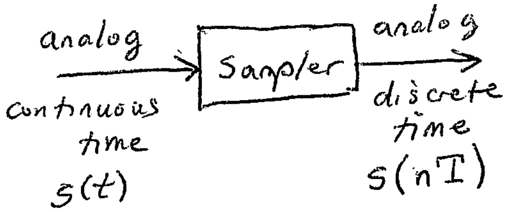
$$\leq \int_{-\infty}^{\infty} \left(|a_0 \delta(\tau)| + |a_1 \delta(\tau - T)| + \dots + |a_{N-1} \delta(\tau - (N-1)T)| \right) d\tau$$

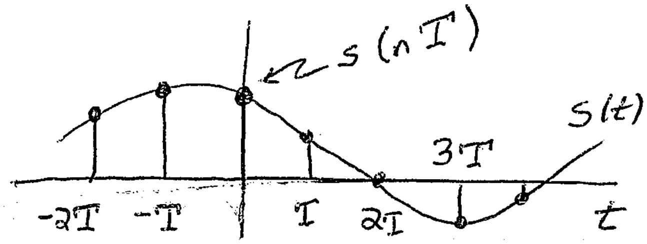
$$= |a_0| + |a_1| + \dots + |a_{N-1}|$$

BIBO stable if $a_0, a_1, \dots, a_{N-1}$ have finite values. K-39

Problem 1.4 Potpourri. *15 points.*

(a) Give an example of a discrete-time analog signal. Please plot your example. *5 points.*

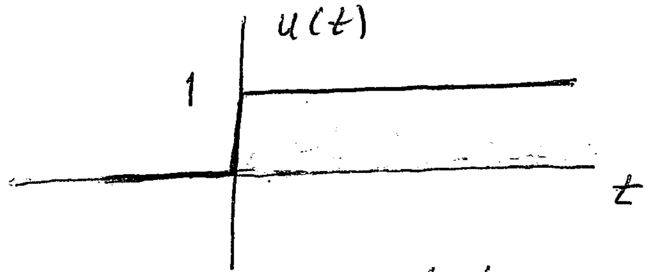




Output of a sampler is sometimes called sampled analog.

(b) Give an example of a continuous-time digital signal. Please plot your example. *5 points.*

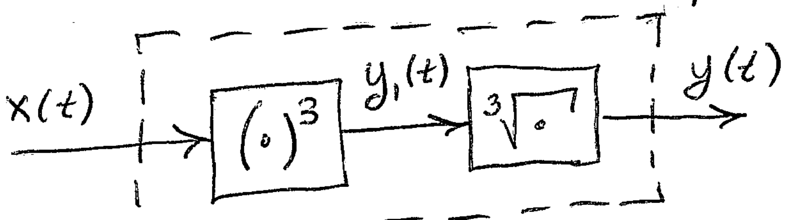
Consider a signal whose amplitude of 1 means "on" and an amplitude of 0 means "off", e.g. modeling of a switch over time.



Switch turns on at $t=0$ and stays on. $u(0)=1$.

(c) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false*: If a system contains a nonlinear subsystem, then the system is always a nonlinear system. *5 points. Please note that writing only true or false will receive zero points.*

False. Here is a counterexample:



Taking the cube of the input is nonlinear.

$$y_1(t) = x^3(t)$$

Scale input by a:

$$y_{1,\text{scaled}}(t) = (a\,x^3(t)) = a^3 x(t) \neq a\,y_1(t) \;\forall a$$

The composite system is $y(t) = x(t)$, which is linear.

Homogeneity: $y_{\text{scaled}}(t) = a\,x(t) = a\,y(t)$

Additivity: $y_{\text{additive}}(t) = x_1(t) + x_2(t) = y_1(t) + y_2(t)$.